CGS602A: Basic Statistics, Data Analysis and Inference Mid-semester exam

Max marks: 80 Time:2 hours

18 Oct. 2020

- 1. Answer all 4 questions.
- 2. You can use calculators.
- 3. Use the SND (standard normal distribution) table where needed.
- 4. Do not collaborate.
- (a) We want to understand the distribution of height in the Indian population. What will be the nature of this distribution? Justify your answer by clearly stating any assumptions you make. In your assumptions stick as closely as possible to approximately what is generally known about the Indian population.
 - (b) Let p be the probability that a head appears in a coin toss. Let the random variable X = number of tosses required to get a head.
 - i. For any x = 1, 2, 3... what is P(X = x)?
 - ii. What is the cdf, $F_X(x) = P(X \le x)$? Try and get a nice closed form expression for $F_X(x)$.
 - iii. Sketch $F_X(x)$ for $-\infty < x < \infty$.

[8,(3,(2,3),4)=20]

2. Consider the figure below which shows a dart board on a wall with area A.



Assume when a dart is thrown it will always hit the wall but may or may not hit the board. The numbers indicate the points earned if the throw lands in the corresponding ring. Also, assume that the circles have the radii 1, 2, 3, 4, 5 in some unit (i.e. r = 5) and the probability of hitting a ring is proportional to its area. Answer the questions below:

- (a) What is the function P(scoring i points) for i = 0..5, where 0 points are given if the throw misses the dart board completely and hits the wall?
- (b) Calculate the conditional probability P(scoring i points | dart board is hit).
- (c) Suppose you are an expert player and never completely miss the dart board. Derive an expression for P(scoring i points), i = 1..5.
- (d) What is the relation between the probability functions in (b) and (c)? Justify briefly.
- (e) Show that the function P(scoring i points) in (c) is indeed a probability function by showing that it satisfies all the Kolmogorov axioms.

[5,5,5,4,6=25]

- (a) Suppose you draw a simple random sample of size 20 from a population whose distribution is not known. Can you claim that the sampling distribution of the mean will be a normal distribution? Justify.
 - (b) Now you draw 100 simple random samples of size 8 each from a normally distributed population. What can you claim about the nature of the sampling distribution of the mean in this case? Justify.
 - (c) There are 6 identical tiles in a box with numbers 1 to 6 written on them. You select a simple random sample of size 2 without replacement. What will be the probability that the mean of a sample that you choose is 3.0.

[4,4,7=15]

- 4. You are a plant biologist working to improve the size of melons. After measuring a sufficiently large number of melons of the new variety you conclude that the melon diameter D is normally distributed with a mean of $\mu = 11.5cm$ and a standard deviation of $\sigma = 1.15cm$.
 - (a) What is the probability that the melon diameter is between 10cm and 13cm?
 - (b) To advertise this as an **improved variety** of melon it is required that at least 80% of the melon diameters be between 10*cm* and 13*cm* and at 90% be greater than equal to 10*cm*. Can you claim that your new variety is an **improved variety**? Justify.
 - (c) If you had made measurements on 5000 melons how many melons will be i) between 10cm and 13cm in diameter, ii) greater than equal to 10cm in diameter iii) less than equal to 13cm in diameter?

[Hint: you will need the tables given for the standard normal distribution for this question.]

[7,7,(2,2,2)=20]

 Table A.3
 Standard Normal Curve Areas (cont.)

 $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998